**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Solution:

Given, the time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. Therefore, X = 60 min - 10 min = 50 min.

Here, the probability that the service manager meet his commitment is

Z = (X-*μ*)*/σ*

> pnorm(50,45,8)

[1] 0.7340145

The probability that the service manager meet his commitment is 0.734.

Now, the probability that the service manager cannot meet his commitment is (1 - 0.734) = 0.267. Option B is the correct answer.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution: Given, the current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6.

1. False Statement. Since the data is normally distributed we can't expect more employees at the processing center are older than 44 than between 38 and 44.
2. True Statement. We see the program for employees under the age of 30

P(Z = (X-*μ)/σ*)

> pnorm(30,38,6)

[1] 0.09121122

Now, we substitute the probability value with the total clerical employees

= 400 \* 0.0912 = 36 approximately.

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Solution:

If *X1* and *X*2 are *iid* normal then *X1* + *X*2 is also normal.

If *X1* and *X*2 are *iid* normal then 2*X1* is also normal.

The variance in 2 *X*1 is more compared to the variance in *X*1 + *X*2

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Solution:

We need to find the Confidence Interval for 99%.

(100±2.58\*20)

(100±51.6)

(48.4, 151.6) is close to option D

Option D is the right answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Solution:

Assuming that $1 = Rs. 45

Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42)

Total Profit ~ N(540, 2252) in rupees

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

We need to find the Confidence Interval for 95%.

(540±1.96\*225)

(99, 981)

B. Specify the 5th percentile of profit (in Rupees) for the company

We have Z score for 5th percentile.

(540-1.645\*225)

(169.875)

Approximately 170.

C. For the First division Profit1 ~ N(5, 32)

we have, (0-5/3) = -1.67

For the Second division Profit2 ~ N(7, 42)

we have, (0-7/4) = -1.75

Hence, we conclude that second division has a larger probability of making a loss in a given year.